

18. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular  $\leftrightarrow$  polar “shortcuts.” In this solution, we employ the “traditional” methods (such as Eq. 3-6).

- (a) The magnitude of  $\vec{a}$  is  $\sqrt{4^2 + (-3)^2} = 5.0$  m.
- (b) The angle between  $\vec{a}$  and the  $+x$  axis is  $\tan^{-1}(-3/4) = -37^\circ$ . The vector is  $37^\circ$  *clockwise* from the axis defined by  $\hat{i}$ .
- (c) The magnitude of  $\vec{b}$  is  $\sqrt{6^2 + 8^2} = 10$  m.
- (d) The angle between  $\vec{b}$  and the  $+x$  axis is  $\tan^{-1}(8/6) = 53^\circ$ .
- (e)  $\vec{a} + \vec{b} = (4 + 6)\hat{i} + ((-3) + 8)\hat{j} = 10\hat{i} + 5\hat{j}$ , with the unit meter understood. The magnitude of this vector is  $\sqrt{10^2 + 5^2} = 11$  m; we rounding to two significant figures in our results.
- (f) The angle between the vector described in part (e) and the  $+x$  axis is  $\tan^{-1}(5/10) = 27^\circ$ .
- (g)  $\vec{b} - \vec{a} = (6 - 4)\hat{i} + (8 - (-3))\hat{j} = 2\hat{i} + 11\hat{j}$ , with the unit meter understood. The magnitude of this vector is  $\sqrt{2^2 + 11^2} = 11$  m, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that  $\vec{a} \perp \vec{b}$ ).
- (h) The angle between the vector described in part (g) and the  $+x$  axis is  $\tan^{-1}(11/2) = 80^\circ$ .
- (i)  $\vec{a} - \vec{b} = (4 - 6)\hat{i} + ((-3) - 8)\hat{j} = -2\hat{i} - 11\hat{j}$ , with the unit meter understood. The magnitude of this vector is  $\sqrt{(-2)^2 + (-11)^2} = 11$  m.
- (j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the  $+x$  direction are  $\tan^{-1}(11/2) = 80^\circ$ , and  $180^\circ + 80^\circ = 260^\circ$ . The latter possibility is the correct answer (see part (k) for a further observation related to this result).
- (k) Since  $\vec{a} - \vec{b} = (-1)(\vec{b} - \vec{a})$ , they point in opposite (antiparallel) directions; the angle between them is  $180^\circ$ .